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RESEARCH ARTICLE

Solving System of Linear Equations with Fuzzy Parameters

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ABSTRACT

The main objective of this paper is to find the positive fuzzy solution of the fully fuzzy linear system and the dual fully fuzzy linear system whose parameters are positive triangular fuzzy numbers, using the method of least squares. Numerical examples to illustrate the use of this method are shown.

Keywords: Fully fuzzy linear system, Dual fully fuzzy linear system, Triangular fuzzy numbers. Method of Least squares.

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1. Introduction

The system of linear equations plays a vital role in various fields, including mathematics, operations research, statistics, physics, social sciences, and engineering. Many applications of linear systems involve parameters and measurements represented by fuzzy numbers rather than crisp numbers. Therefore, it is essential to develop a mathematical model and numerical procedures to solve linear systems with fuzzy parameters. When all the parameters in a linear system are fuzzy numbers, it is referred to as a fully fuzzy linear system. In this context, the coefficient matrix, the unknown vector, and the right-hand side column vector are all fuzzy numbers. A linear system of this form $\widetilde{D}_1 \otimes \tilde{x} = \widetilde{D}_2 \otimes \tilde{x} \oplus \tilde{e}$ and $\widetilde{D}_1 \otimes \tilde{x} \oplus e_1 = \widetilde{D}_2 \otimes \tilde{x} \oplus \tilde{e}_2$ is known as a dual fully fuzzy linear system.

The concept of fuzzy numbers and fuzzy arithmetic operations was first introduced by Zadeh [5, 6]. Dehghan et al. [1] have solved the $n \times n$ fully fuzzy linear system with triangular fuzzy numbers using Cramer's rule, the Gauss elimination method, the LU decomposition method, and the linear programming approach. Nasseri et al. [2] proposed an LU decomposition method for a fully fuzzy linear system with triangular fuzzy numbers. Radhakrishnan et al. [3, 4] solved the $m \times n$, $(m \geq n)$ fully fuzzy linear system with trapezoidal fuzzy numbers using the QR decomposition method and simplex method.

In this paper, the fully fuzzy linear system of the form $\widetilde{D} \otimes \tilde{x} = \tilde{e}$, the dual fully fuzzy linear systems of the form $\widetilde{D}_1 \otimes \tilde{x} = \widetilde{D}_2 \otimes \tilde{x} \oplus \tilde{e}$ and $\widetilde{D}_1 \otimes \tilde{x} \oplus e_1 = \widetilde{D}_2 \otimes \tilde{x} \oplus \tilde{e}_2$ are considered and the positive solution is obtained by using the method of least squares, where $\widetilde{D}, \widetilde{D}_1, \widetilde{D}_2$ are $m \times n$, $(m \geq n)$ fuzzy matrices consisting of positive triangular fuzzy numbers, the unknown vector $\tilde{x}$ is a fuzzy vector consisting of n positive triangular fuzzy numbers, and the constants $\tilde{e}, \tilde{e}_1, \tilde{e}_2$ are fuzzy vectors consisting of m positive triangular fuzzy numbers.

2. Preliminaries

2.1. Definition

Let X be a universal set. Then, we defined the fuzzy subset $\widetilde{D}$ of X by its membership function $\mu_{\widetilde{D}} : X \to [0, 1]$ which assigns to each element $x \in X$ a real number $\mu_{\widetilde{A}}(x)$ in the interval $[0, 1]$, where the function value of $\mu_{\widetilde{D}}(x)$ represents the grade of membership of x in $\widetilde{D}$. A fuzzy set $\widetilde{D}$ is written as $\widetilde{D} = \{(x, \mu_{\widetilde{D}}(x)), x \in X, \mu_{\widetilde{D}}(x) \in [0, 1]\}$.

2.2. Definition

A fuzzy number $\widetilde{D} = (a, b, c)$ is said to be a triangular fuzzy number, if its membership function is given by

$$\boldsymbol{\mu_{\widetilde{D}}(x)} = \begin{cases} \mathbf{1} - \frac{\boldsymbol{a-x}}{\boldsymbol{b}}, & \boldsymbol{a - b \leq x \leq a, b} > 0 \\ \mathbf{1} - \frac{\boldsymbol{x-a}}{\boldsymbol{c}}, & \boldsymbol{a \leq x \leq a + c, c} > 0 \\ \mathbf{0}, & \boldsymbol{otherwise} \end{cases}$$

2.3 Definition

A triangular fuzzy number $\widetilde{D} = (a, b, c)$ is said to be positive (negative) if and only if $a - b \geq 0$ $(a + c \leq 0)$.

2.4. Definition

Two triangular fuzzy numbers $\widetilde{D} = (a, b, c), \widetilde{E} = (d, e, f)$ are equal if and only if $a = d, b = e, c = f$

2.5. Definition

Let $\widetilde{D} = (a, b, c), \widetilde{E} = (d, e, f)$ are two triangular fuzzy numbers then
$\widetilde{D} \oplus \widetilde{E} = (a + d, b + e, c + f)$

2.6. Definition

Let $\widetilde{D} = (a, b, c), \widetilde{E} = (d, e, f)$ are two triangular fuzzy numbers then
$(i)\ \widetilde{D} \ominus \widetilde{E} = (a - d, b + f, c + e)$
$(ii)\ \widetilde{D} \ominus \widetilde{D} = (a - a, b + c, c + b) = (0, b + c, b + c) \neq (0,0,0) = \tilde{0}$
$(iii)\ \widetilde{E} \ominus \widetilde{E} = (d - d, e + f, e + f) = (0, e + f, e + f) \neq (0,0,0) = \tilde{0}$

2.7. Definition

Let $\widetilde{D} = (a, b, c) > 0, \widetilde{E} = (d, e, f) > 0$ are two triangular fuzzy numbers then$\widetilde{D} \otimes \widetilde{E} = (ad, ae + bd, af + cd)$

2.8. Definition

A matrix $\widetilde{D} = (\widetilde{d}_{ij})$ is called a fuzzy number matrix or fuzzy matrix, if each element of $\widetilde{D}$ is a fuzzy number. A matrix $\widetilde{D}$ is a positive fuzzy matrix ($\widetilde{D} \geq 0$), if each element of $\widetilde{A}$ is positive.

2.9. Definition

Let $\widetilde{D} = \left(\widetilde{d}_{ij}\right)$ *and* $\widetilde{E} = \left(\tilde{e}_{ij}\right)$ be two $m \times p$ and $p \times n$ fuzzy matrices. We define $\widetilde{D} \otimes \widetilde{E} = \widetilde{F} = \left(\widetilde{f}_{ij}\right)$ which is the $m \times n$ matrix where $\widetilde{f}_{ij} = \sum_{k=1,2\dots.n}^{\oplus} \widetilde{d}_{ik} \otimes \tilde{e}_{kj}$

2.10. Definition

A vector $\widetilde{X} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)^T$ is called a fuzzy vector, if element of $\widetilde{X}$ are a fuzzy numbers.

2.11. Definition

The $m \times n$ fuzzy linear system

$$\left(\widetilde{d}_{11} \otimes \tilde{x}_1\right) \oplus \left(\widetilde{d}_{12} \otimes \tilde{x}_2\right) \oplus \dots \oplus \left(\widetilde{d}_{1n} \otimes \tilde{x}_n\right) = \tilde{e}_1$$
$$\left(\widetilde{d}_{21} \otimes \tilde{x}_1\right) \oplus \left(\widetilde{d}_{22} \otimes \tilde{x}_2\right) \oplus \dots \oplus \left(\widetilde{d}_{2n} \otimes \tilde{x}_n\right) = \tilde{e}_2$$
$$\vdots$$
$$\left(\widetilde{d}_{m1} \otimes \tilde{x}_1\right) \oplus \left(\widetilde{d}_{m2} \otimes \tilde{x}_2\right) \oplus \dots \oplus \left(\widetilde{d}_{mn} \otimes \tilde{x}_n\right) = \tilde{e}_m$$

This can be written as $\widetilde{P} \otimes \tilde{x} = \tilde{a}$. Where the coefficient matrix $\widetilde{D} = \left(\widetilde{d}_{ij}\right),\ i = 1\ to\ m,\ j = 1\ to\ n,$ is a triangular fuzzy matrix and $\widetilde{d} = \left(\widetilde{d}_1, \widetilde{d}_2, \dots, \widetilde{d}_m\right)^T$ is a triangular fuzzy number vector and the $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)^T$ is the unknown triangular fuzzy number vector. It is called a fully fuzzy linear system.

2.12. Definition

We may represent $m \times n$ fuzzy matrix $\widetilde{D} = \left(\widetilde{d}_{ij}\right)_{m \times n}$, such that $\widetilde{D}_{ij} = \left(a_{ij}, b_{ij}, c_{ij};\ \right)$ with the new notation $\widetilde{D} = (A, B, C)$, where $A = \left(\boldsymbol{a_{ij}}\right), \boldsymbol{B} = \left(\boldsymbol{b_{ij}}\right),\ \boldsymbol{C} = \left(\boldsymbol{c_{ij}}\right),$ are three $m \times n$ crisp matrices.

3. The method of Least Squares

3.1. Least-Squares Solutions [7]

Suppose that $Ax = b$ does not have a solution.

The best approximate solution of $Ax = b$ is called the least-squares solution.

That is, "best approximate solution" to an inconsistent matrix equation $Ax = b$.

3.2. Definition [7]

Let A be an $m \times n$ matrix and let b be a vector in R^m. A least-squares solution of the matrix equation $Ax = b$ is a vector $\hat{x}$ in R^n such that $dist(b, A\hat{x}) \leq dist(b, Ax)$ for all other vectors x in R^n.

3.3. Theorem [7]

Let A be an $m \times n$ matrix and let b be a vecotr in R^m. The least squares solutions of $Ax = b$ are the solution of the matrix equation $A^T Ax = A^T b$.

4. Method of finding the positive solution of the fully fuzzy linear system

4.1. Consider the fully fuzzy linear system $\widetilde{\boldsymbol{D}} \otimes \tilde{\boldsymbol{x}} = \tilde{\boldsymbol{e}}$ (Here all the parameter are triangular fuzzy numbers)

Where $\widetilde{D} = (A, B, C) \geq 0, \tilde{x} = (x, y, z) \geq 0;\ \tilde{e} = (f, g, h) \geq 0$

$(A, B, C) \otimes (x, y, z) = (f, g, h)$

$(Ax, Ay + Bx, Az + Cx) = (f, g, h)$ $\quad$ *using definition* 2.7.

$$(A^T Ax, A^T(Ay + Bx), A^T(\ Az + Cx)) = (A^T f, A^T\ g, A^T h)$$

Using definition 2.4., we have

$A^T Ax = A^T f$

$A^T(Ay + Bx) = A^T\ g$

$A^T(\ Az + Cx) = A^T h$

The above equation can be written as

$A^T Ax = A^T f$
$A^T Ay = A^T(g - Bx)$

$A^T Az = A^T(h - Cx)$

Therefore,

$x = (A^T A)^{-1}\ (A^T f)$

$y = (A^T A)^{-1}[A^T(g - Bx)]$

$z = (A^T A)^{-1}[A^T(h - Cx)]$

Where $(A^T A)^{-1} \neq 0$

4.2. Consider the dual fully fuzzy linear system $\widetilde{D}_1 \otimes \tilde{x} = \widetilde{D}_2 \otimes \tilde{x} \oplus \tilde{e}$ (Here all the parameter are triangular fuzzy numbers)

Where $\widetilde{D}_1 = (A_1, B_1, C_1) \geq 0, \widetilde{D}_2 = (A_2, B_2, C_2) \geq 0, \tilde{x} = (x, y, z) \geq 0;\ \tilde{e} = (f, g, h) \geq 0$

$$(A_1, B_1, C_1) \otimes (x, y, z) = (A_2, B_2, C_2) \otimes (x, y, z) \oplus (f, g, h)$$

$(A_1x, A\ {}_1y + B_1x, A_1z + C_1x) = (A_2x, A\ {}_2y + B_2x, A_2z + C_2x) \oplus (f, g, h)$ using definition 2.7.

$(A_1x, A\ {}_1y + B_1x, A_1z + C_1x) = (A_2x + f, A\ {}_2y + B_2x + g, A_2z + C_2x + h)$ using definition 2.5.

Using definition 2.4., we have

$A_1x = A_2x + f$

$A\ {}_1y + B_1x = A\ {}_2y + B_2x + g$

$A_1z + C_1x = A_2z + C_2x + h$

The above equation can be written as

$(A_1 - A_2)x = f$

$(A_1 - A_2)y + (B_1 - B_2)x = g$

$(A_1 - A_2)z + (C_1 - C_2)x = h$

Let us take $A_1 - A_2 = A, B_1 - B_2 = B, C_1 - C_2 = C$, then

$Ax = f$

$Ay = g - Bx$

$Az = h - Cx$

Using the method for computing a least-squares solutions of $Ax = f$ are the solutions of the matrix equation $A^TAx = A^Tf$

Therefore, $x = (A^TA)^{-1}\ (A^Tf)$

Similarly we have

$y = (A^TA)^{-1}[A^T(g - Bx)]$

$z = (A^TA)^{-1}[A^T(h - Cx)]$

Where $(A^TA)^{-1} \neq 0$

4.3. Consider the dual fully fuzzy linear system $\widetilde{D}_1 \otimes \tilde{x} \oplus \tilde{e}_1 = \widetilde{D}_2 \otimes \tilde{x} \oplus \tilde{e}_2$ (Here all the parameter are triangular fuzzy numbers)

Where $\widetilde{D}_1 = (A_1, B_1, C_1) \geq 0, \widetilde{D}_2 = (A_2, B_2, C_2) \geq 0,$ $\tilde{x} = (x, y, z) \geq 0,$

$\tilde{e}_1 = (f_1, g_1, h_1) \geq 0, \tilde{e}_2 = (f_2, g_2, h_2) \geq 0$

$$(A_1, B_1, C_1) \otimes (x, y, z) \oplus (f_1, g_1, h_1) = (A_2, B_2, C_2) \otimes (x, y, z) \oplus (f_2, g_2, h_2)$$

$(A_1x, A\ {}_1y + B_1x, A_1z + C_1x) \oplus (f_1, g_1, h_1) = (A_2x, A\ {}_2y + B_2x, A_2z + C_2x) \oplus (f_2, g_2, h_2)$ using definition 2.7.

$(A_1x + f_1, A\ {}_1y + B_1x + g_1, A_1z + C_1x + h_1) = (A_2x + f_2, A\ {}_2y + B_2x + g_2, A_2z + C_2x + h_2)$ using definition 2.5.

Using definition 2.4., we have

$A_1x + f_1 = A_2x + f_2$

$A\ {}_1y + B_1x + g_1 = A\ {}_2y + B_2x + g_2$

$A_1z + C_1x + h_1 = A_2z + C_2x + h_2$

The above equation can be written as

$(A_1 - A_2)x = f_2 - f_1$

$(A_1 - A_2)y + (B_1 - B_2)x = g_2 - g_1$

$(A_1 - A_2z + (C_1 - C_2)x = h_2 - h_1$

Let us take $A_1 - A_2 = A, B_1 - B_2 = B, C_1 - C_2 = C$, $f_2 - f_1 = f, g_2 - g_1 = g,$ $h_2 - h_1 = h$, then

$Ax = f$

$Ay = g - Bx$

$Az = h - Cx$

Using the method for computing a least-squares solutions of $Ax = f$ are the solutions of the matrix equation $P^TPx = p^Ta$

Therefore, $x = (A^TA)^{-1}\ (A^Tf)$

Similarly we have

$y = (A^TA)^{-1}[A^T(g - Bx)]$

$z = (A^TA)^{-1}[A^T(h - Cx)]$

Where $(A^TA)^{-1} \neq 0$

5. Numerical examples

5.1. consider the following fully fuzzy linear system

$(5,3,3) \otimes \tilde{x} \oplus (6,2,2) \otimes \tilde{y} = (39,28,28)$

$(7,1,1) \otimes \tilde{x} \oplus (6,3,3) \otimes \tilde{y} = (45,28,28)$

$(7,2,2) \otimes \tilde{x} \oplus (4,2,2) \otimes \tilde{y} = (37,25,25)$

Where $\tilde{x}, \tilde{y} \geq 0$

Solution:

$$\begin{bmatrix} (5,3,3) & (6,2,2) \\ (7,1,1) & (6,3,3) \\ (7,2,2) & (4,2,2) \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = \begin{bmatrix} (39,28,28) \\ (45,28,28) \\ (37,25,25) \end{bmatrix}$$

$A = \begin{pmatrix} 5 & 6 \\ 7 & 6 \\ 7 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 2 \\ 1 & 3 \\ 2 & 2 \end{pmatrix}, C = \begin{pmatrix} 3 & 2 \\ 1 & 3 \\ 2 & 2 \end{pmatrix}, f = \begin{pmatrix} 39 \\ 45 \\ 37 \end{pmatrix}, g = \begin{pmatrix} 28 \\ 28 \\ 25 \end{pmatrix}, h = \begin{pmatrix} 28 \\ 28 \\ 25 \end{pmatrix}$

$x = (A^T A)^{-1} (A^T f)$

$y = (A^T A)^{-1} [A^T (g - Bx)]$

$z = (A^T A)^{-1} [A^T (h - Cx)]$

Solving the above system of equations we have

$\tilde{x} = (3,1,1);\ \tilde{y} = (4,1,1)$

5.2. consider the following fully fuzzy linear system

$(10,3,3) \otimes \tilde{x} \oplus (5,2,2) \otimes \tilde{y} = (5,3,3) \otimes \tilde{x} \oplus (6,2,2) \otimes \tilde{y} \oplus (11,4,4)$

$(12,1,1) \otimes \tilde{x} \oplus (6,3,3) \otimes \tilde{y} = (7,1,1) \otimes \tilde{x} \oplus (6,3,3) \otimes \tilde{y} \oplus (15,5,5)$

$(16,2,2) \otimes \tilde{x} \oplus (8,2,2) \otimes \tilde{y} = (7,2,2) \otimes \tilde{x} \oplus (4,2,2) \otimes \tilde{y} \oplus (43,13,13)$

Where $\tilde{x}, \tilde{y} \geq 0$

Solution:

$A_1 = \begin{pmatrix} 10 & 5 \\ 12 & 6 \\ 16 & 8 \end{pmatrix}, B_1 = \begin{pmatrix} 3 & 2 \\ 1 & 3 \\ 2 & 2 \end{pmatrix}, C_1 = \begin{pmatrix} 3 & 2 \\ 1 & 3 \\ 2 & 2 \end{pmatrix}$

$A_2 = \begin{pmatrix} 5 & 6 \\ 7 & 6 \\ 7 & 4 \end{pmatrix}, B_2 = \begin{pmatrix} 3 & 2 \\ 1 & 3 \\ 2 & 2 \end{pmatrix}, C_2 = \begin{pmatrix} 3 & 2 \\ 1 & 3 \\ 2 & 2 \end{pmatrix}, f = \begin{pmatrix} 11 \\ 15 \\ 43 \end{pmatrix}, g = \begin{pmatrix} 4 \\ 5 \\ 13 \end{pmatrix}, h = \begin{pmatrix} 4 \\ 5 \\ 13 \end{pmatrix}$

$$A = A_1 - A_2 = \begin{pmatrix} 5 & -1 \\ 5 & 0 \\ 9 & 4 \end{pmatrix}, B = B_1 - B_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, C = C_1 - C_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$x = (A^T A)^{-1} (A^T f)$

$y = (A^T A)^{-1} [A^T (g - Bx)]$

$z = (A^T A)^{-1} [A^T (h - Cx)]$

Solving the above system of equations we have

$\tilde{x} = (3,1,1);\ \tilde{y} = (4,1,1)$

5.3. consider the following fully fuzzy linear system

$(10,3,3) \otimes \tilde{x} \oplus (5,2,2) \otimes \tilde{y} \oplus (12,10,14) = (5,3,3) \otimes \tilde{x} \oplus (6,2,2) \otimes \tilde{y} \oplus (23,14,18)$

$(12,1,1) \otimes \tilde{x} \oplus (6,3,3) \otimes \tilde{y} \oplus (14,8,12) = (7,1,1) \otimes \tilde{x} \oplus (6,3,3) \otimes \tilde{y} \oplus (29,13,17)$

$(16,2,2) \otimes \tilde{x} \oplus (8,2,2) \otimes \tilde{y} \oplus (16,6,10) = (7,2,2) \otimes \tilde{x} \oplus (4,2,2) \otimes \tilde{y} \oplus (59,19,23)$

Where $\tilde{x}, \tilde{y} \geq 0$

Solution:

$A_1 = \begin{pmatrix} 10 & 5 \\ 12 & 6 \\ 16 & 8 \end{pmatrix}, B_1 = \begin{pmatrix} 3 & 2 \\ 1 & 3 \\ 2 & 2 \end{pmatrix}, C_1 = \begin{pmatrix} 3 & 2 \\ 1 & 3 \\ 2 & 2 \end{pmatrix}, f_1 = \begin{pmatrix} 12 \\ 14 \\ 16 \end{pmatrix}, g_1 = \begin{pmatrix} 10 \\ 8 \\ 6 \end{pmatrix}, h_1 = \begin{pmatrix} 14 \\ 12 \\ 10 \end{pmatrix}$

$A_2 = \begin{pmatrix} 5 & 6 \\ 7 & 6 \\ 7 & 4 \end{pmatrix}, B_2 = \begin{pmatrix} 3 & 2 \\ 1 & 3 \\ 2 & 2 \end{pmatrix}, C_2 = \begin{pmatrix} 3 & 2 \\ 1 & 3 \\ 2 & 2 \end{pmatrix}, f_2 = \begin{pmatrix} 23 \\ 29 \\ 59 \end{pmatrix}, g_2 = \begin{pmatrix} 14 \\ 13 \\ 19 \end{pmatrix}, h_2 = \begin{pmatrix} 18 \\ 17 \\ 23 \end{pmatrix}$

$A = A_1 - A_2 = \begin{pmatrix} 5 & -1 \\ 5 & 0 \\ 9 & 4 \end{pmatrix}, \quad B = B_1 - B_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, C = C_1 - C_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

$f = f_2 - f_1 = \begin{pmatrix} 11 \\ 15 \\ 43 \end{pmatrix}, g = g_2 - g_1 = \begin{pmatrix} 4 \\ 5 \\ 13 \end{pmatrix}, h = h_2 - h_1 = \begin{pmatrix} 4 \\ 5 \\ 13 \end{pmatrix}$

$x = (A^T A)^{-1} (A^T f)$

$y = (A^T A)^{-1} [A^T (g - Bx)]$

$z = (A^T A)^{-1} [A^T (h - Cx)]$

Solving the above system of equations we have

$\tilde{x} = (3,1,1);\ \tilde{y} = (4,1,1)$

6. Conclusion

The $m \times n$, $(m \geq n)$ fully fuzzy linear system is converted into three different $m \times n$ crisp linear systems, and then the positive solution is obtained by using the method of least squares. Also given the procedure to solve the dual fully fuzzy linear systems of the form $\widetilde{D}_1 \otimes \tilde{x} = \widetilde{D}_2 \otimes \tilde{x} \oplus \tilde{e}$ and $\widetilde{D}_1 \otimes \tilde{x} \oplus e_1 = \widetilde{D}_2 \otimes \tilde{x} \oplus \tilde{e}_2$. Radhakrishnan et al. [3, 4] solved the $m \times n$, $(m \geq n)$ fully fuzzy linear system with trapezoidal fuzzy numbers using the QR decomposition method and simplex method, but these procedures are very lengthy, and the time consumption is very high when compared to method of least squares.

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