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RESEARCH ARTICLE

NANO g^* α- NORMAL SPACES IN NANO TOPOLOGICAL SPACES

ADINATHA C. UPADHYA[1] AND MAMATA M. K[2]

[1]Associate Professor, Department of Mathematics, S.R.F.G.Constituent College, Belagavi-590017. Karnataka, India
[2]Lecturer, Department of Mathematics, G.S.Science College, Belagavi-590011. Karnataka. India.

ABSTRACT

The aim of this paper, we introduce a new class of nano normal spaces in a nano topological space. We obtain the relationships of such normal spaces and present some properties and establish various preservation theorems.

Keywords: Nano open, nano α-open, nano $g^*α$ -normal space, nano $g^*α$-continuous

1.Introduction

In the year 1971 Viglino[10] who defined semi normal space and the concept of almost normal spaces was introduced by Signal and Arya [9] and they demonstrated that a space is normal only if it is also both a semi-normal and an almost normal space. In 2013 L.Thivagar [5,6] introduced the notion of nano topological spaces for a subset X of a universe that is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano-open sets. He has also studied nano closure and nano interior of a set. Later he was introduced and studied the certain weak forms of nano-open sets namely nano α-open sets, nano pre-open sets etc. The characterizations of mildly nano gb-normal spaces were studied by Arul Mary and Arockiarani I [1].

2. PRELIMINARIES

DEFINITION 2.1[8] :

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its is denoted by $L_R(X)$. That is

$L_R(X) = \bigcup_{x\in u} \{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and its is denoted by $U_R(X)$.

That is $U_R(X) = \bigcup_{x\in u} \{R(x): R(x) \cap X \neq \emptyset\}$, where R(x) denotes the equivalence class determined by x

(iii) The boundary region of X with respect to R is the set of all subjects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

DEFINITION 2.2 [5]:

Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ Where $X \subseteq U$. Then, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\emptyset \in \tau_R(X)$,
(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is , $\tau_R(X)$ is a topology on U called the nanotopology on U with respect to X. We call $(U,\tau_R(X))$ as the nanotopological space. The elements $\tau_R(X)$ are called as nano-open sets.

REMARK 2.3 [5]:

If $\tau_R(X)$ is the nano topology on U with respect to X, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

DEFINITION 2.4 [5]:

If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano -open subsets of A and it is denoted by NInt(A).

DEFINITION 2.5 [45]:

If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano closure of A is defined as the intersection of all nano -closed sets containing A and it is denoted by NCl(A).

DEFINITION 2.6 [6]:

If $(U, \tau_R (X))$ is a nano topological space and $A \subseteq U$. Then A is said to be
(i) Nano semi -open if $A \subseteq$ NCl(NInt(A))
(ii) Nano pre- open (briefly nano p-open) if $A \subseteq$ NInt(NCl(A))
(iii) Nano α - open if $A \subseteq$ NInt(NInt(NCl(A)))
NSO(U, X), NSPO(U,X) and NαO(U,X) respectively we denote the families of all nano semi-open, nano pre-open and nano α-open subsets of $(U, \tau_R (X))$. Let $(U, \tau_R (X))$ be a nano topological space and $A \subseteq U$. A is said to be nano semi-closed, nano pre-closed and nano α-closed if its complement is respectively nano semi-open, nano pre-open and nano α-open.

DEFINITION 2.7 [2] :

A subset A of $(U, \tau_R (X))$ is called nano generalized closed set (briefly Ng-closed) if Ncl(A) $\subseteq$ V whenever $A \subseteq V$ and V is nano open in $(U, \tau_R (X))$.

DEFINITION 2.8 [3] :

A subset A of $(U, \tau_R (X))$ is called nano generalized α closed set (briefly Ngα -closed) if Nαcl(A) $\subseteq$ V whenever $A \subseteq V$ and V is nano open in $(U, \tau_R (X))$.

DEFINITION 2.9 :

A subset A of $(U, \tau_R (X))$ is said to be a α -neighborhood of u, if there exists a nano-open set V such that $u \in V \subseteq A$

DEFINITION 2.10 :

A function f: $(U, \tau_R (X)) \rightarrow (V, \tau_{R'} (Y))$ is said to be (1) nano continuous [6] if the inverse image every nano open set V is nano open in U. (2) almost nano α-irresolute if for each u in U and each nano α-neigjbourhood N of f(u) in V, $N\alpha Cl(f^{-1}(V))$ is a nano α-neighbourhood of u in U. (3) nano Mα-closed (nano Mα-open), if f(A) is nano α-closed(resp. nano α-open) set in V for each nano α-closed (resp. nano α-open) set A in U. (4) nano gα closed if f(F) is nano gα-closed set in V for every nano closed subset F of U.

LEMMA 2.11:

A function f: $(U, \tau_R (X)) \rightarrow (V, \tau_{R'} (Y))$ is weakly nano open nano continuous function, then f is nano Mα -open and nano R-map.

2. NANO g*α- NORMAL SPACES

DEFINITION 3.1:

Let (U,τR(X)) be a Nano topological spaces. A subset A of (U,τR(X)) is called Nano g*α-closed set if Ncl(A)⊆V Where A⊆V and V is Nano g-open.

DEFINITION 3.1 :

A nano topological space $(U, \tau_R (X))$ is said to be nano g*α- normal spaces if for any pair of disjoint nano closed sets A and B of U there exist nano g*α- pen sets V and W of U such that A M and B N.

EXAMPLE 3.2:

Let U = {a,b,c}with U/R = {{a}, {b, c}}. Then the nano topology, τ_R (X)= {U, , {a}, {b,c}}. Hence the only pair of disjoint closed subsets of $(U, \tau_R (X))$ is {b, c}, {a}. Also {a},{b,c} are nano g*α-open sets such {a} {a} and {b,c} {b,c}.

We have the following characterization of nano g*α - normal spaces.

THEOREM 3.4:

For a nano topological spaces $(U, \tau_R (X))$ and $(V, \tau_{R'} (X))$ the following are equivalent.

(i) U is nano g*α- normal space.
(ii) for every pair of nano open sets M and N whose union is U, there exist nano g*α-closed sets A and B such that A M and B N and A B=U.
(iii) for every nano closed set H and every nano open set B containing A, there exists anano g*α- open set M such that H M N g*α Cl(M) K.

PROOF:

(i) ⟹ (ii): Let M and N be a pair of nano open sets in a nano g*α- normal space U such that $M \cup N = U$. Then, U\M and U \N are disjoint nano closed sets. Since U is nano g*α- normal, there exist disjoint nano g*α -open sets M_1 and N_1 such that $U\backslash M \subseteq M_1$ and $U \backslash N \subseteq N_1$. Let $A = U\backslash M_1$ and $B= U \backslash N_1$. Then A and B are nano g*α-closed sets such that $A\subseteq M$ and $B \subseteq N$ and $A \cup B=U$.

(ii) ⟹ (iii): Let H be a nano closed set and K be a nano open set containing H. such that $H \subseteq K$. Then, U\H and K are nano open sets whose union is U. Then by (ii), there exist nano g*α -closed sets P_1 and P_2 such that $P_1 \subseteq U\backslash H$ and $P_2 \subseteq K$ and $P_1 \cup P_2 = U$. Then $H \subseteq U\backslash P_1$ and $U\backslash K \subseteq U\backslash P_2$ and $(U\backslash P_1) \cap (U\backslash P_2)= \phi$. Let $M= U\backslash P_1$ and $N= U\backslash P_2$. Then M and N are disjoint nano g*α -open sets such that $H \subseteq M \subseteq U\backslash N \subseteq K$. Since U\N is nano g*α -closed, then we have N g*αCl(M) $\subseteq$ U\N and $H \subseteq M \subseteq$ N g*αCl(M) $\subseteq$ K.

(iii) ⟹ (i): Let H_1 and H_2 are any two disjoint nano closed sets of U. Put $K=U\backslash H_2$, then $H_2 \cap K= \phi$, $H_1 \subseteq K$, where K is an nano open set. Then, by (iii), there exists a nano g*α -open set M of U such that $H_1 \subseteq M \subseteq$ N g*αCl(M) $\subseteq$ K. It follows that H_2 = U\N

$g^*\alpha Cl(M)$ = N, say, Then N is nano $g^*\alpha$ -open and $M \cap N = \phi$. Hence, H_1 and H_2 are separated by nano $g^*\alpha$-open sets M and N. Therefore, U is nano $g^*\alpha$ -normal.

Preservation theorems of nano g^* α-normal spaces.

DEFINITION 3.5:

A function f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is called strongly nano $g^*\alpha$-open if $f(M) \in Ng^*\alpha O(V)$ for each $M \in Ng^*\alpha O(U)$.

DEFINITION 3.6:

A function f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is called strongly nano $g^*\alpha$-closed if $f(M) \in Ng^*\alpha C(V)$ for each $M \in Ng^*\alpha C(U)$

THEOREM 3.7:

A function f : $(U, \tau R(X)) \rightarrow (V, \tau R'(Y))$ is called strongly nano $g^*\alpha$ -closed if and only if for each subset B in V and for each nano $g^*\alpha$ -open set M in U containing $f^{-1}(B)$, there exist a nano $g^*\alpha$ -open set N containing B such that $f^{-1}(N) \subset M$

PROOF:

Suppose that f is strongly nano $g^*\alpha$-closed. Let B be a subset of V and $M \in N\ g^*\alpha O(U)$ containing $f^{-1}(B)$. Put N = V − f (U − M), then N is a nano $g^*\alpha$-open set of V such that $B \subset N$ and $f^{-1}(N) \subset M$. Conversely let K be any nano $g^*\alpha$-closed set of U. Then $f^{-1}(V - f(K)) \subset U - K$ and $U - K \in N\ g^*\alpha O(U)$. There exists a nano $g^*\alpha$-open set N of V such that $V - f(K) \subset N$ and $f^{-1}(N) \subset U - K$. Therefore, we have $f(K) \supset V - N$ and $K \subset f^{-1}(V - N)$. Hence, we obtain f (K) = V − N and f(K) is nano $g^*\alpha$-closed in V. This shows that f is strongly nano $g^*\alpha$-closed.

THEOREM 3.8:

If f :$(U, \tau R(X)) \rightarrow (V, \tau R'(Y))$ is a strongly nano $g^*\alpha$-closed continuous function from a nano $g^*\alpha$-normal space U on to a space V, then V is nano $g^*\alpha$-normal.

PROOF:

Let K_1 and K_2 are disjoint nano closed sets in V. Then $f^{-1}(K_1)$ and $f^{-1}(K_2)$ are nano closed sets in V. Since U is nano $g^*\alpha$-normal then there exists disjoint nano $g^*\alpha$-open sets M and N such that $f^{-1}(K_1) \subset M$ and $f^{-1}(K_2) \subset N$. Then there exists nano $g^*\alpha$-open sets A and B such that $K_1 \subset A$, $K_2 \subset B$, $f^{-1}(A) \subset M$ and $f^{-1}(B) \subset N$. Also, A and B are disjoint. Thus, V is nano $g^*\alpha$-normal.

DEFINITION 3.9:

A function f: $(U, \tau R(X)) \rightarrow (V, \tau R'(Y))$ is called almost nano $g^*\alpha$- irresolute if for each u in U and each nano $g^*\alpha$-neighbourhood N of f (u), $Ng^*\alpha$ -cl $(f^{-1}(N))$ is a nano $g^*\alpha$-neighbourhood of u.

LEMMA 3.10:

For a function f: $(U, \tau R(X)) \rightarrow (V,\tau R'(Y))$ the following statements are equivalent.

(i) f is almost nano $g^*\alpha$-irresolute.

(ii) $f^{-1}(N) \subset N\ g^*\alpha\text{-int}(N\ g^*\alpha\text{-cl}(f^{-1}(N)))$ for every $N \in Ng^*\alpha O(V)$.

THEOREM 3.11:

A function f :$(U, \tau R(X)) \rightarrow (V, \tau R'(Y))$ almost nano $g^*\alpha$-irresolute if and only if $f(N\ g^*\alpha\text{-cl}(M)) \subset Ng^*\alpha\text{-cl}(f(M))$ for every $M \in N\ g^*\alpha O(U)$.

PROOF:

Let $M \in N\ g^*\alpha O(U)$. Suppose $V \in/\ N\ g^*\alpha$ -cl(f(M). Then there exists $N \in N\ g^*\alpha O(V, v))$ such that $N \cap f(M) = \phi$. Hence, $f^{-1}(N) \cap M = \phi$ .Since $M \in N\ g^*\alpha O(U)$. We have $N\ g^*\alpha\text{-int}(N\ g^*\alpha\text{-cl}(f^{-1}(N))) \cap N\ g^*\alpha\text{-cl}(M) = \phi$. Then by lemma [3.10], $f^{-1}(N) \cap N\ g^*\alpha\text{-cl}(M) = \phi$ and hence $N \cap f(Ng^*\alpha\text{-cl}(M)) = \phi$. This implies that $V \notin f(N\ g^*\alpha\ cl(M))$. Conversely if $N \in Ng^*\alpha O(V)$ then $P = U/N\ g^*\alpha\text{-cl}(f^{-1}(N)) \in Ng^*\alpha O(U)$. By hypothesis, $f(Ng^*\alpha\text{-cl}(P)) \subset Ng^*\alpha$ --cl(f (P)) and hence, $U/N\ g^*\alpha\text{-int}(N\ g^*\alpha - cl(f^{-1}(N))) = Ng^*\alpha\text{-cl}(P) \subset f^{-1}(N\ g^*\alpha\text{-cl}(f(P))) \subset f^{-1}N\ g^*\alpha - cl(f(U)/f^{-1}(N)) \subset f^{-1}Ng^*\alpha cl(V/N) \subset f^{-1}(V/N) = U/f^{-1}(N)$. Therefore $f^{-1}(N) \subset Ng^*\alpha\text{-int}(Ng^*\alpha\text{-cl}(f^{-1}(N))$. By Lemma [3.10], f is almost nano $g^*\alpha$-irresolute.

THEOREM 3.12:

If f: $(U, \tau R(X)) \rightarrow (V, \tau R'(Y))$ is a strongly nano $g^*\alpha$-open continuous almost nano $g^*\alpha$-irresolute function from a nano $g^*\alpha$-normal space U onto a space V. Then V is nano $g^*\alpha$-normal space.

PROOF:

Let A be a nano closed set of K and B be a nano open set containing A. Then by continuity of f, $f^{-1}(A)$ is nano closed and $f^{-1}(B)$ is nano open set of U such that $f^{-1}(A) \subset f^{-1}(B)$. As U is nano $g^*\alpha$-normal, there exists a nano $g^*\alpha$-open set M in U such that $f^{-1}(A) \subset M \subset Ng^*\alpha\text{-cl}(M) \subset f^{-1}(B)$ by Theorem 2.4. Then $f(f^{-1}(A)) \subset f(M) \subset f(N\ g^*\alpha cl(M)) \subset f(f^{-1}(B))$. Since f is strongly nano $g^*\alpha$-open almost nano $g^*\alpha$-irresolute surjection, we obtain $A \subset f(M) \subset N\ g^*\alpha\text{-cl}(f(M) \subset B$. Then again Theorem 3.4 the space V is a nano $g^*\alpha$- normal.

4. ALMOST NANO g^* α– NORMAL SPACES

DEFINITION 4.1:

A nano topological spaces $(U, \tau R(X))$ is said to be almost nano $g^*\alpha$-normal if for each nano $g^*\alpha$ -closed set A and nano regular closed set B such that $A \cap B = \phi$, there exist disjoint nano $g^*\alpha$ -open sets M and N such that $A \subset M$ and $B \subset N$.

DEFINITION 4.2:

A nano topological spaces (U, $\tau R(X)$) is said to be quasi nano $g^*\alpha$ -closed if f (A) is nano $g^*\alpha$ -closed in V for each A ∈ $Ng^*\alpha$ C(U).

THEOREM 4.3:

For a nano topological spaces (U, $\tau R(X)$) the following statements are equivalent:

(i) U is almost nano $g^*\alpha$ -normal.

(ii) For every pair of nano sets A and B, one of which is nano $g^*\alpha$-open and the other is nano regular open whose union is U, there exist nano nano $g^*\alpha$-closed sets H and K such that H ⊂ A and K ⊂ B and H ∪ K =U.

(iii) For every nano $g^*\alpha$-closed set H and nano regular open set K containing H, there exists a nano $g^*\alpha$ -open set N such that H ⊂ B ⊂ $Ng^*\alpha$ cl(B) ⊂ K.

PROOF:

(i) ⇒ (ii) Let A and B be a pair of nano open sets in a nano $g^*\alpha$ -normal space U such that U = A ∪ B. Then U − A and U − B are two disjoint nano $g^*\alpha$ -closed sets. Since U is nano $g^*\alpha$-normal there exist nano $g^*\alpha$-open sets A_1 and such B_1, such that U − A ⊂ A_1 and U− B ⊂ B_1. Let H = U − A_1, K = U − B_1. Then H and K are nano $g^*\alpha$ -closed sets such that H ⊂ A and K ⊂ B and H ∪ K = U .

(ii)⇒ (iii) Let A be a nano $g^*\alpha$ -closed set and B be a nano $g^*\alpha$ -open set containing A. The U − A and B are nano $g^*\alpha$ -open sets whose union is U. Then by (ii), there exist nano $g^*\alpha$ -closed sets W_1 and W_2 such that W_1 ⊂ U − A and W_2 ⊂ B and W_1 ∪ W_2 = U. Then A ⊂ U − W_1 and U − B ⊂ W_2 and (U − W_1) ∩ (U − W2) = ϕ. Let X = U − W_1 and Y = U − W_2. Then A and B are disjoint nano $g^*\alpha$ -open sets such that A ⊂ U − Y ⊂ B. As U − Y is nano $g^*\alpha$-closed set, we have N $g^*\alpha$ cl(X) ⊂ U − Y and A ⊂ X ⊂ N $g^*\alpha$cl(X) ⊂ B.

(iii)⇒ (i): Let A_1 and A_2 be any two disjoint nano $g^*\alpha$ -closed sets of U. Put B = U − A_2, then A_2 ∩ B = ϕ. A_1 ⊂ B where B is a nano $g^*\alpha$ -open sets. Then by (iii), there exists a nano $g^*\alpha$ -open set X of U such that A_1 ⊂ U − N $g^*\alpha$cl(X) = Y, then Y is nano $g^*\alpha$-open and X ∩ Y = ϕ. Hence A_1 and A_2 are separated by nano $g^*\alpha$open sets X and Y. Therefore U is nano $g^*\alpha$-normal.

THEOREM 4.4:

If f : (U, $\tau R(X)$) → (V, $\tau R'(Y)$) is a nano $g^*\alpha$ -continuous, quasi nano $g^*\alpha$-closed surjection and U is nano $g^*\alpha$-normal, then V is normal.

PROOF:

Let W_1 and W_2 be any disjoint nano $g^*\alpha$ -closed sets of V. Since f is nano $g^*\alpha$-continuous, $f^{-1}(W_1)$ and $f^{-1}(W_2)$ are disjoint nano $g^*\alpha$-closed sets of U. Since U is nano $g^*\alpha$-normal, there exist disjoint N_1 and N_2 ∈ N $g^*\alpha$O(V), such that $f^{-1}(W_1)$ ⊂ Ni for i = 1, 2. Put Qi = V − f (U − Ni then Qi is nano $g^*\alpha$ -open in V, W_2 ⊂ Qi and f^{-1}(Qi) ⊂ Ni for i = 1, 2. Since N_1 ∩ N_2 = ϕ and f is surjective. We have Q_1 ∩ Q_2 = ϕ. This shows that V is nano $g^*\alpha$-normal

5. MIDLY NANO g^* α – NORMAL SPACES

DEFINITION 5.1 :

A nano topological (U, τ_R (X)) is said to mildly nano $g^*\alpha$ -normal if for every pair of disjoint nano regular closed set A and B of U, there exist disjoint nano $g^*\alpha$ -open sets M and N such that A ⊂ M and B ⊂ N.

THEOREM 5.2:

For a nano topological space (U, τ_R (X)) the following are equivalent.

(i) U is mildly nano $g^*\alpha$ - normal space.

(ii) for every pair of nano regular open sets M and N, whose union is U, there exists nano $g^*\alpha$-closed sets A and B such that A ⊂ M and B ⊂ N and A ∪ B=U.

(iii) for any nano regular closed set A and every nano regular open set B containing A, there exists a nano $g^*\alpha$ -open set M such that A ⊂ M ⊂ N $g^*\alpha$-cl(M) ⊂ B.

(iv) for every pair of disjoint nano regular closed sets A and B, there exists a nano $g^*\alpha$ -open sets M and N such that A ⊂ M and B ⊂ N, N $g^*\alpha$-cl(M) ∩ N $g^*\alpha$-cl(N)=ϕ.

PROOF:

(i) ⟹ (ii): Let M and N be a pair of nano regular open sets in a mildly nano $g^*\alpha$- normal space U such that M ∪ N = U. Then, U\M and U \N are disjoint nano regular closed sets. Since U is mildly nano $g^*\alpha$-normal, there exist disjoint nano $g^*\alpha$-open sets M_1 and N_1 such that U\M ⊂ M_1 and U \N ⊂ N_1. Let A = U\ M_1 and B= U \N_1. Then A and B are nano $g^*\alpha$-closed sets such that A ⊂ M and B ⊂ N and A ∪ B=U.

(ii) ⟹ (iii): Let A be a nano regular closed set and B be a nano regular open set containing A such that A ⊂ B. Then, U\A and B are nano regular open sets whose union is U. Then by (ii), there exist nano $g^*\alpha$-closed sets P_1 and P_2 such that P_1 ⊂ U\A and P_2 ⊂ B and P_1 ∪ P_2 = U. Then A ⊂ U\ P_1 and U\B ⊂ U\ P_2 and (U\ P_1) ∩ (U\ P_2)= ϕ . Let M= U\ P_1 and N= U\ P_2. Then M and N are disjoint nano $g^*\alpha$-open sets such that A ⊂ M ⊂ U\N ⊂ B. Since U\N is nano $g^*\alpha$-closed, then we have N $g^*\alpha$-cl(M) ⊂ U\N and A ⊂ M ⊂ N $g^*\alpha$-cl(M) ⊂ B.

(iii) ⟹ (iv): Let A and B be two nano regular closed sets such that A ∩ B =ϕ. Then, A ⊂ U\B which is nano regular open. Therefore, there exists a $g^*\alpha$-open set M such that A ⊂ M ⊂ N $g^*\alpha$-cl(M) ⊂ U\B. Again, N is a $g^*\alpha$-open set containing the nano regular closed set A. Therefore, there is a $g^*\alpha$-open set N such that A ⊂ N ⊂ N $g^*\alpha$-cl(N) ⊂ M. Let U\ N $g^*\alpha$-cl (M) = V. Then, A ⊂ M and B ⊂ N, N $g^*\alpha$-cl(M) ∩ N $g^*\alpha$ -cl(V)=ϕ.

(iv) ⟹ (i): Obvious.

THEOREM 5.3:

If f: (U, τ_R (X)) → (V, $\tau_{R'}$ (Y)) is strongly nano g*α -open nano R-map and almost nano g*α -irresolute function from a mildly nano g*α -normal space U onto a space V, then V is mildly nano g*α -normal.

PROOF:

Let A be nano regular closed set and B be a nano regular open set containing A. Then by nano R-map of f, f^{-1} (A) is a nano regular closed set contained in the nano regular open set f^{-1} (B) . Since U is mildly nano g*α -normal, there exists a nano g*α -open set N such that f^{-1} (A) ⊂ N⊂ N g*α -cl(N) f^{-1} (B) by Theorem 4.3. As f is strongly nano g*α -open and an almost nano g*α -irresolute surjection, it follows that f(N) N g*α O(V) and A ⊂ f(N) ⊂ N g*α-cl(f(N)) ⊂ B. Hence V is mildly nano g*α -normal.

THEOREM 5.4:

If f: (U, τ_R (X)) → (V, $\tau_{R'}$ (Y)) is nano R-map, strongly nano g*α -closed function from a mildly nano g*α -normal space U onto a space V, then V is mildly nano g*α -normal.

PROOF:

Let K_1 and K_2 be disjoint nano closed sets in V. Then by nano R-map of f, f^{-1} (K_1) and f^{-1} (K_2) are disjoint nano closed sets of U. Since U is nano g*α -normal, there exists a nano g*α -open sets M and N such that f^{-1} (A) ⊂ M and f^{-1} (B) ⊂ N. By Theorem 4.3, there exists nano g*α -open sets A and B such that K_1⊂ A, K_2 ⊂ B, f^{-1} (A) ⊂ M and , f^{-1} (B) ⊂ N. Also, A and B are disjoint. Thus, V is nano g*α-normal.

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